

١-٧ الأعداد التخيلية Imaginary numbers

$$1 - \sqrt{-1} = -i \quad \leftarrow$$

تمارين ١-٧

(١) اكتب كلاً مما يأتي في أبسط صورة:

د $\sqrt{-81} + \sqrt{-16}$

ج $\sqrt{-90}$

ب $\sqrt{\frac{36}{81} - 1}$

ا $\sqrt{144 - 1}$

د $\sqrt{81 \times -1} + \sqrt{16 \times -1}$
 $= \sqrt{81} \times \sqrt{-1} + \sqrt{16} \times \sqrt{-1}$
 $= 9i + 4i = 13i$

ا $\sqrt{144 - 1} = \sqrt{-1 \times 144} = \sqrt{-1} \times \sqrt{144} = 12i$

ب $\sqrt{\frac{36}{81} - 1} = \sqrt{\frac{36}{81} - \frac{81}{81}} = \sqrt{\frac{36 - 81}{81}} = \sqrt{\frac{-45}{81}} = \sqrt{-1} \times \sqrt{\frac{45}{81}} = \frac{\sqrt{45}}{9}i = \frac{3\sqrt{5}}{9}i = \frac{\sqrt{5}}{3}i$

ج $\sqrt{-90} = \sqrt{9 \times 10 \times -1} = 3\sqrt{10}i$

$\boxed{i^2 = -1}$

(٢) بسّط كلاً مما يأتي:

د $\frac{5-i}{2+6i}$

ج $\sqrt{\frac{100 - 4i^2}{4}}$

ب $\frac{9-i - (2-i)^2}{2}$

ا $-5i^2 + 3i^4$

$\frac{5-i}{2+6i} \times \frac{2-6i}{2-6i} = \frac{(5-i)(2-6i)}{4-36i^2} = \frac{10-30i-2i+6i^2}{4+36} = \frac{10-32i-6}{40} = \frac{4-32i}{40} = \frac{1-8i}{10}$

ا $-5i^2 + 3i^4 = -5(-1) + 3(1) = 5 + 3 = 8$

ب $\frac{9-i - (2-i)^2}{2} = \frac{9-i - (4-4i+i^2)}{2} = \frac{9-i - (4-4i-1)}{2} = \frac{9-i - (3-4i)}{2} = \frac{9-i-3+4i}{2} = \frac{6+3i}{2} = 3 + \frac{3}{2}i$

ج $\sqrt{\frac{100 - 4i^2}{4}} = \sqrt{\frac{100 - 4(-1)}{4}} = \sqrt{\frac{100 + 4}{4}} = \sqrt{\frac{104}{4}} = \sqrt{26} = \sqrt{26}$

د $\frac{5-i}{2+6i} = \frac{5-i}{2+6i} \times \frac{2-6i}{2-6i} = \frac{(5-i)(2-6i)}{4-36i^2} = \frac{10-30i-2i+6i^2}{4+36} = \frac{10-32i-6}{40} = \frac{4-32i}{40} = \frac{1-8i}{10}$

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